Calculation of Magnetic Field Disturbance Produced by Electric Railway

by

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Abstract

Magnetic field disturbance by leakage current of electric railways is a serious problem for magnetic observations. We can get an exact solution in various situations of the route of railway, distributions of power substations and electric railcars by the method developed herein. The developed method yields a smaller estimating magnetic field disturbance compared with the old method.

1. Introduction

It is known that the leakage current from a DC electric railcar forms magnetic fields in the surrounding environment. Unwanted noises occur from these magnetic fields, causing interference to the observation of natural magnetic fields. Because the noises spread two-dimensionally, they travel over a long distance without decaying and cause interference at remote locations. They have complex waveforms and it is technically impossible to remove them under the circumstances.* If the railway track could be completely insulated from the ground and if the current supplied to drive the electric railcar could be made to flow only in a closed circuit from a power substation to a feeder line to an electric railcar, and then to a railway track, the noises would not propagate so long a distance even if a DC electric railcar is used. Because the railway track and the feeder line are installed close to one another, the effect of the same amount of current flowing in them cancel each other. Therefore, at a remote location the circuit is regarded as if it carried no currents. In reality, however, a considerable amount of leakage current flows into the ground, because the railway track is not completely

insulated from the ground. Not only does the leakage current itself form magnetic fields, but it also causes the amount of current through the railway track to differ from the amount of current through the feeder line and, as a result, magnetic fields form around the railway track and the feeder line. The imperfect insulation thus gives rise to two types of noise: the noise formed by the leakage current and that formed by the current through the railway track and the feeder.

To observe natural magnetic fields, we must quantitatively analyze the effects of magnetic field disturbances caused by the leakage current from a DC electric railcar. In this study, we conducted experiments, made field surveys, performed model calculations, and attempted to verify the effects of magnetic field disturbances caused by the leakage current from the DC electric railcar.

Inoue (1979) performed model calculations to simulate the effects of the noises from magnetic fields around the Joban Line by placing a DC electric railcar at different positions on the railway track. In his approach to the model calculations, we noted some shortcomings as pointed out in the following:

^{*} In the case of an AC electric railcar, the skin effect prevents the leakage current from flowing deep into the ground, and the frequency of an AC current is higher than the frequencies at which natural magnetic fields are normally observed. Therefore, the leakage current from the AC electric railcar does not form magnetic fields, and interfering noises do not occur.

- (1) When deriving the amount of current supplied by each power substation, a certain fixed leak rate was used, and the model calculations were performed based on the leak rate.
- (2) When obtaining solutions from the results of analyses, boundary conditions were not established, and the scenario that two or more electric railcars run simultaneously on the railway track was not considered. Solutions for each electric railcar were simply superposed.
- (3) Bouncing of the potential distributed across the railway track back to power substations was not considered.

Yanagihara (1977) developed approximate calculations for physically precise solutions and estimated disturbance fields by assuming a simple model of a railway. He obtained satisfactory results by comparing calculated values with measured values.

By further developing the approach taken by Yanagihara (1977), we studied magnetic field disturbances, created a new model for simulating real conditions as closely as possible to the extent practical and physically consistent, and performed calculations based on several scenarios. This paper describes the results of our study and calculations. We expect that our model would give a clear basis for discussion, reducing the number of unconsidered parameters (such as the non unifomity of the subterranean structure.)

2. Calculation Method

(1) How to calculate the leakage current

As the number of power substations increases, the amount of current that each power substation supplies to one electric railcar varies depending on the distance between that power substation and the electric railcar. Generally, power substations near the electric railcar supply a larger amount of current than those at some distance from the electric railcar.

We studied how to calculate the leakage current on the assumption that the amount of current used by one electric railcar is always the same and the positions of the electric railcars, as well as the positions of each power substation, are known. As shown in Figure 1, a railway segment starting at point A and ending at point B was assumed. A coordinate "s" was established along this railway segment. The direction from A to B was defined as a positive direction. A current flowing in the railway track to be measured at point "*s*" was defined as " $J_r(s)$," and the leakage current from this railway track was defined as "*i*(*s*)." "*i*(*s*)" can be expressed as follows:

$$i(s) = -\frac{dJ_r(s)}{ds} \tag{1}$$

Because the railway track was not considered isoelectric, it was thought that the potential is distributed along the railway track. This distributed potential was defined as V(s). With the railway track resistance per unit length and the leakage resistance defined as " R_r " and " η " respectively, the relationships between V(s), $J_r(s)$ and i(s) are expressed as follows:

$$\frac{dV(s)}{ds} = -R_r J_r(s) \tag{2}$$

$$V(s) = \eta i(s) \tag{3}$$

Based on above three equations, the following ordinary differential equation can be formulated regarding $J_r(s)$:

$$\frac{d^2 J_r(s)}{ds^2} = \left(\frac{R_r}{\eta}\right) J_r(s) \tag{4}$$

A general solution to this differential equation can be obtained using equation (5) below:

$$J_r(s) = C_1 \exp(\alpha s) + C_2 \exp(-\alpha s)$$
 (5)

In this equation, $\alpha = \sqrt{(R_r/\eta)}$ (6)

 C_1 and C_2 in equation (5) are constants to be determined by boundary conditions.

If there is more than one power substation in the A-B railway segment and if more than one electric railcar runs in this railway segment, the number of boundaries increases, and C_1 and C_2 must be determined for each subsegment delimited by boundaries. For example, if the total number of power substations and electric railcars in the A-B segment is 10, the number of boundaries including that of the A-B segment is 12. This means that the A-B segment has 11 subsegments. Therefore, C_1 and C_2 must be obtained for each of these 11 subsegments. The following discussion assumes that neither electric railcars nor power substations are set at points A and B.



rail current $J_r(s)$ at railway s, and direction of current.

substation, fieder line and railway.

(7)

g.3. Leakage current i(s)ds, at railway s, and position vector \vec{r} .

One subsegment delimited by the boundaries $s = s_k$ and $s = s_{k+1}$ inside the A-B segment as shown in Figure 2 was established. The railway currents that flow to the boundary from the left (A side) were defined as J_k and J_{k+1} , and the current that branches to the feeder line at s_k was defined as I_k . Based on equation (5), the railway current inside the subsegment delimited by s_k and s_{k+1} can be calculated as follows:

$$J_r(s) = \frac{J_{k+1} \sinh\{\alpha(s-s_k)\} - (J_k - I_k) \sinh\{\alpha(s-s_{k+1})\}}{\sinh \alpha l_k}$$

where $l_k = s_{k+1} - s_k$ Providing that the total number of power substations

and electric railcars is n:

If s_k is a power substation, $I_k > 0$

If s_k is an electric railcar, $I_k < 0$ $(1 \le k \le n)$

Note that $J_0 = 0$ (start point A) and $J_{n+1} = 0$ (end point B).

In I_k (k = 1, 2, ..., n), the number of electric railcars (n-m) is a known constant. I_k for power substations (m) and all J_k (n) are unknowns. These unknowns (n+m) are calculated based on the following three conditions:

Condition 1: Current conservation law

The total amount of current supplied by power substations is equal to the amount of electric-car current, and the current from power substations and the current in the electric railcar flow in opposite directions. Therefore, the following equation can be formulated:

$$\sum_{i=1}^{n} I_i = 0 \tag{8}$$

Condition 2: Conditions regarding potential V(s) at boundaries

Potential V(s) must be continuous at each boundary $s_k (k = 1, 2, ..., n)$.

$$V(s_k-0)=V(s_k+0)$$

Therefore,

$$-S_{k-1}J_{k-1} + (T_{k-1}+T_k)J_k - S_kJ_{k+1} + S_{k-1}I_{k-1} - T_kI_k = 0$$
(9)

in which

$$S_{k} = \frac{1}{\sinh\{\alpha(s_{k+1} - s_{k})\}}$$

$$T_{k} = \frac{1}{\tanh\{\alpha(s_{k+1} - s_{k})\}}$$
(10)

Condition 3: Kirchhoff's second law

Kirchihoff's second law is applied to the circuit in which a current flows out of one power substation, enters another power substation via the feeder line, and returns to the original power substation via the railway track. Two power substations, $s = s_1$ and s_k , were assumed. The voltage from s_1 to s_k on the railway track is expressed as follows:

$$V(s_1) - V(s_k)$$

The voltage from a power substation (s_1) to the feeder line, then to another power substation (s_k) is expressed as follows:

$$-(E_0-I_1R_s)+\sum_{j=1}^{k-1}(\sum_{i=1}^j I_i)R_f+(E_0-I_kR_s)$$

Where the power substation voltage is defined as E_0 (constant), the internal resistance of a power substation as R_f , and the resistance of the feeder line as R_f .

Because the former voltages are equal to each other, applying equation (1), (3) and (7) to these, the following equation can be derived.

$$R_{s}(I_{k}-I_{l}) - \sum_{j=1}^{k-1} (\sum_{i=1}^{j} I_{i})R_{f} - \alpha\eta \{S_{1}T_{1} - T_{1}(J_{1}-I_{1}) - S_{k}J_{k+1} + T_{k}(J_{k}-I_{k})\} = 0$$
(11)

This relationship is applied to the power substation on the left end (defined as $s = s_1$) and other power substations (*m*-1).

The numbers of equations obtained based on the three conditions shown on the previous page are 1, n, and m-1, respectively. The total number is (n+m). Therefore, the unknown number matches the number of equations, and the simultaneous equation can be solved. I_k and J_k calculated this way are substituted into equation (7) to obtain $J_r(s)$. $J_r(s)$ obtained this way is substituted into equation (1) to obtain the leakage current i(s).

(2) How to calculate the amplitude of magnetic field disturbances

Although the leakage current spreads radially underground, the effect of the leakage current on magnetic fields can be defined as the effect of a semi-infinite line current that flows perpendicular to the ground. As Yanagihara pointed out in 1977, magnetic field (H) formed by the leakage current from the A-B segment and currents through the railway track and the feeder line at external point Pcan be calculated as follows:

$$H = \int_{s_0}^{s_{n+1}} \left(\frac{\overrightarrow{e_z \times r}}{r^2} \right) i(s) ds + \int_{s_0}^{s_{n+1}} \left(\frac{\overrightarrow{ds \times r}}{r^3} \right) \left[J_f(s) + J_r(s) \right]$$

(12)

in which $\vec{e_s}$ is the unit vector in the vertical direction, \vec{r} is the position vector from point *s* in the A-B segment to point *P*, and $J_f(s)$ is the feeder current.

The first term represents magnetic field formed by the leakage current, and the second term represents a field formed by currents through the railway track and the feeder line. We performed the calculations using the method of successive approximation based on Simpson's rule.

3. Calculating the Amplitude of Magnetic Field Disturbances

3.1 Comparison with the Values from the Generally Used Model

(1) Amount of current supplied by power substations

To simulate real conditions as closely as possible, it is necessary to extend the length of the railway track and to establish more than one power substation. It is thought that power substations near the electric railcar supply a larger amount of current than those at some distance from the electric railcar. Although quantitative calculations were made to clarify the relationships between the amount of current and the distance between power substations and the electric railcar, the appropriateness of the established preconditions for calculations was questioned, as mentioned in Chapter 1. Introduction (the study made by Inoue in 1979). In this study, we compared the results of calculations made using the new model with those of calculations made using the generally used model under the same conditions as when Inoue performed the calculations.

Figure 4 shows the amount of current supplied by each power substation in a case where an electric railcar was set at the extreme left of the segment (0 km) and ten power substations were set at 1 km intervals. In the case of the generally used model, the amount of current supplied by each power substation was determined only by the resistance of the feeder line. In the case of the new model, the amount of current supplied by each power substation decreased rapidly as the distance from the electric railcar increased, because not only the resistance of the feeder line but also the resistance of the railway track and the ground leakage resistance were taken into consideration. However, it is inappropriate to conclude from this that the magnetic field is disturbed only by the current around the electric railcar. For example, the amount of current that leaks to the ground and then returns again to the railway track will increase if power substations are set at long intervals. Therefore, it should be noted that there are cases in which the amount of current being supplied by power substations will increase though they are set apart at distant intervals.

(2) Relationships between the position of the electric railcar and the amplitude of magnetic field disturbances

Figure 5 shows the position of Kakioka relative to stations on the Joban Line, assuming that the line is DC electrified as far as Tsuchiura Station. In performing the calculations, the railway track of the Joban Line was limited to the segment from Tsuchiura Station to Abiko Station, and this segment was approximated by a line graph, as shown in Figure 5. Figures 6 and 7 show the amplitudes of horizontal magnetic field disturbances calculated under the same conditions as in Figure 5. The intervals between power substations shown in these figures are for power substations for the railway section as far as Fujishiro Station. In the railway section from Fujishiro Station to other stations in the up-line direction of the Joban Line, one power substation is established each at Fujishiro, Toride and Abiko.

Figure 6 shows the values calculated using the generally used model, and Figure 7 shows those



Fig.4. Comparison of the supply current through power substations between the new model and the old one.

This figure shows currents through power substations in the condition that ten power substations are ordered at 1 Km intervals.



Fig.6. Horizontal magnetic field disturbance at KAKIOKA as a function of position of one electric car calculated by the old model. (After CHIGIKI KANSOKUSHO MONDAI KENKYUKAI HOKOKUSHO by IBARAKI-KEN)

calculated using the new model. Positions of the electric railcar are shown along the abscissa axis with Tsuchiura Station as the starting point. The amplitudes of horizontal magnetic field disturbances measured at Kakioka are shown along the ordinate axis. Because calculations were performed by changing the interval between power substations, each curve represents the state of disturbance relative to each specific interval.

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Fig.5. Map of KAKIOKA and Joban Line. The railway from TSUCHIURA to ABIKO is approximated by segments of straight line.



Fig.7. Horizontal magnetic field distubance at KAKIOKA calculated by the new model.

For a railway segment in which a specific interval between power substations is known, the total leakage current can be calculated as follows:

$$\int_{s_i}^{s_{i+1}} i(s)ds = \int_{s_i}^{s_{i+1}} -\{dJ_r(s)/ds\}ds = J_r(s_i) - J_r(s_{i+1})$$
(13)

It should be noted that in this equation J_r is not always uniform throughout the railway track; specifically J_r differs largely in segments $s_i(1, 2, ..., n)$. Because $J_r(s_i)$ in the segment (s_i, s_{i+1}) is different from $J_r(s_i)$ in the segment (s_{i-1}, s_i) , different J_r values must be used as integrands for each segment when performing integrations on the leakage current. The above equation represents rail current variations, and the longer the interval between power substations, the longer the rail length and, as a result, the larger the amount of the leakage current becomes. Therefore, the tendency should be noted that the amplitude of the magnetic field disturbances increases as the interval between power substations increases. This tendency is demonstrated by the results of calculations using both models.

What should be noted about the results of calculations using the new model is that the amplitude of horizontal magnetic disturbances becomes small when the electric railcar is at a point about 10 km from a power substation. We termed this point a singular point.

In performing calculations using both old and new models, only one electric railcar was used. We found that the amount of the leakage current was large at points near the electric railcar and that the leakage current returned to the railway track at distant points from the electric railcar. If the railway track ran almost in a straight line as viewed from the observation point, we found that magnetic field disturbances caused by the leakage current near the electric railcar and those caused by the returning current near the observation point cancelled each other out after the electric railcar traveled a certain distance. We also found that components perpendicular to the railway track dominated other components and that the orientation of these components was reversed at the above mentioned singular point where the amplitude of horizontal magnetic field disturbances dropped markedly. If the railway track is straight and if the observation point is located on an extension of the straight line, there

should be a position of the electric railcar where the amplitude of the magnetic field disturbances becomes zero because the horizontal components of magnetic field disturbances is perpendicular to the railway track. To verify the correctness of this assumption, we used a simple arrangement of the railway track and the observation point shown in Figure 8, and performed the calculations.

Figure 9 is a graph showing the amplitude of the magnetic field disturbances relative to the positions of the electric railcar. As is apparent from this figure, there is the singular point where the horizontal components of magnetic field disturbances become extremely small, as in Figure 7. If the observation point is on an extension of the railway track, magnetic field disturbances consist of only the components perpendicular to the railway track; they do not contain vertical components at all. The railway track was divided into three segments: a 2-km long segment before and after the electric railcar, which was defined as segment B, a segment near the observation point, which was defined as segment C, and a segment far from the observation point, which was defined as segment A. Calculations were performed to verify the specific contributions of these segments to the amplitude of the magnetic field disturbances. Figure 10 is a graph plotted based on the results of the calculations.

Because the leakage current dominates in segment B and the returning current dominates in segments A and C, the orientation of magnetic field disturbances was reversed. The amplitude of the magnetic field disturbances in segment B decreased moderately as the electric railcar was going away from the observation point (as the source of the leakage current was going away from the observation point).

The current that leaked near the electric railcar returned to a near railway track, not to a far railway track. Therefore, in the A and C segments, the amplitude of the magnetic field disturbances was larger at points near the electric railcar than at distant points from it, on the condition that the observation point was at some distance from the railway track.







Fig.9. Horizontal magnetic field disturbance in the case shown by Fig.8.

The amplitude of the magnetic field disturbances in segment A decreased as the electric railcar went away from the observation point. This is because the length of the segment became short and the point near the electric railcar where the amplitude of the magnetic field disturbances was largest in segment A also went away from the observation point. In segment C, although the length of the segment became long as the electric railcar went away from the observation point, the position near the electric railcar where the amplitude of the magnetic field disturbances was largest in segment C went away and, as a result, the amplitude of the magnetic field disturbances did not change noticeably. In this case, the amplitude magnetic field disturbance became zero when the electric railcar was at a point about 13 km from the end of the railway track. When the electric railcar traveled further beyond this 13-km point, the orientation of the magnetic field disturbances was reversed.



Fig.10. Horizontal magnetic field disturbance produced by each segment or railway. Segments A, B and C are shown in Fig.8. Be careful that the direction of magnetic field disturbance produced by

segment B is reverse to those by segments A and C.

3.2 Comparison with the Results of Field Experiments

It is inappropriate to discuss whether the new model has achieved an improvement over the generally used model only by comparing both calculation results. Furthermore, there is no available data on the results of a comparison between measured values and the values calculated using the generally used model. In this study, we compared measured values and the values calculated using the generally used model in order to verify the effectiveness of the new model.

We measured the amplitudes of magnetic field disturbances caused by the effects of the DC electrified segment on the JR Joban Line (to the southwest of an AC - DC switching power substation, which is located between Toride Station and Fujishiro Station) at observation points shown in Figure 11 during the period from Jan. 7 to Jan. 9, 1983. Figure 12 shows the state of observed magnetic field disturbances. Figure 13 shows a comparison between measured values and calculated values.



Fig.11. Map of observation points for field experiment and Joban-Line.

In performing the calculations, the railway track was approximated to a line graph, and contributions of each straight portion were totaled. Two electric railcars were set near Matsudo Station, and one electric railcar was set near Toride Station, based on the timetable.

The magnetic field in observation point Nos. 1, 2, 4 and 5 and observation point Nos. 3, 6, 7 and 8 were measured on different dates. When Nos. 1, 2, 4 and 5 were measured, it rained, so the leakage resistance might be different from the other day's. In observation point 3, 6, 7 and 8, the amplitude of the magnetic field disturbances could not be detected because the observation point was too far from the railway track. Therefore, only observation point Nos. 1, 2, 4 and 5 were used to compare them with calculated values.

Although it was thought that a current of thousands of amperes is supplied to the electric railcar, an exact ampere value could not be measured. Because the amplitude of the magnetic field disturbances is proportional to the amount of current supplied to the electric railcar, calculated values were evaluated at each observation point based on the values obtained by normalizing the amplitude of the magnetic field disturbances at the observation point No. 1 with the calculated amplitude values (measured values/ calculated values).

Concerning the orientation of magnetic field disturbances, it was verified that all measured values were in good agreement with calculated values. As shown in Figure 13, the amplitude of horizontal magnetic field disturbances is almost the same at all observation points, except for at the observation point No. 1, where the amplitude was slightly larger. It was thought that satisfactory results had been obtained regarding all vertical components of magnetic field disturbances.

Vertical components of a magnetic field, which are produced by horizontal components of an electric current, flow through the rail and the feeder line, and the route of current flow is fixed. Considering that horizontal components of a magnetic field are produced by the leakage current, we assumed that in performing the calculations, they would flow in the vertical direction. However, if the subterranean electric conductivity is nonuniform, this assumption does not stand. If it is nonuniform, it was thought that the observation point near the railway track, such as the observation point No.1, would be most strongly affected, while distant observation points where the leakage current could hardly reach would be minimally affected.

3.3 Distribution of Magnetic Field Disturbances in a Simple Arrangement of Observation Points and the Railway Track

The results of calculations using the new model were in good agreement with measured values, as described in section 3.2. Using the new model and a simple arrangement of observation points and the railway track, we performed the calculations to verify the two-dimensional distribution of magnetic field disturbances, which was considered to be the first attempt of its kind in Japan.

A straight railway track was established, power substations were set at equal intervals, and electric railcars were randomly assigned on the track, as shown in Figure 14. In Figure 14, the contour lines distribution, which represents the total magnetic force of magnetic field disturbances, is also shown. In this simple arrangement, the amplitude of the magnetic field disturbances was about 0.15 nT at a point close to the end of the railway track and it was 0.2 to 0.5 nT around the mid-point of the railway track, when



Fig.12. Magnetic field disturbance produced by Joban-Line.

Magnetic field disturbances at No. 1, 2, 4, 5 and at No.3, 6, 7, 8 were observed at different date. Dashed lines show the assumed records without magnetic field disturbance. The difference between line and dashed line shows magnitude of magnetic field disturbance produced by the electric railway.



Fig.13. Comparison of magnetic field disturbance between observed values and calculated values. Bar graphs refer left ordinate, line graphs refer right ordinate. Line graphs show the ratio of observed value and calculated one which is normalized by such ratio at No.1 observation point, so the ratio at No.1 is exactly 1.



Fig.14. Distribution of magnetic total force disturbance in the simple case such as this figure.

measured at a location 30 km from the railway track. This state of amplitude distribution was thought to be associable with where each electric railcar is located on the railway track.

Sharply curved contour lines are due to mutual canceling out of the effects from the two lower electric railcars and one upper electric railcar shown in Figure 14. It is estimated that as the number of electric railcars increases, the distribution of magnetic field disturbances around the railway track becomes complicated.

4. Conclusion

We found that the amplitude of the magnetic field disturbances calculated with the new model was generally smaller than that calculated with the operational model of the Railway Technical Research Institute. Our model can be applied to various situations flexibly, keeping the strictness of the assumption of uniformity of subterranean electric conductivity. In all these calculations, calculated values were in good agreement with measured values. On the other hand, we noted the following difficulties:

- It was difficult to identify η (leakage resistance), a parameter of the new model. This parameter was unstable and varied greatly depending on the weather. It may also vary depending on railway laying conditions. Therefore, it is inappropriate to assume that this parameter is uniform through the whole railway track.
- · It was assumed that the leakage current would flow over an infinite distance in the vertical direction on the condition that the subterranean electric conductivity was uniform. However, the subterranean distribution of currents is not really uniform if the subterranean electric conductivity is distributed anisotropically. For example, if the electric conductivity at a depth more than a few kilometers from the ground surface is small, the leakage current stops there and does not flow deeper.

These difficulties, however, can be somewhat overcome by using the following techniques:

- Concerning η , equations (1) through (6) can be used as is if specific values are assigned to each interval between power substations.
- · Concerning the route of the leakage current flow, because the leakage current generally flows only a few kilometers deep, it can be assumed that the leakage current first flows vertically a few kilometers, then turns its direction, and continues to flow at the same depth along the same route as the railway track. Under this assumption, the difference between the amount of currents in the railway track and that of currents in the feeder line would decrease further due to the current flowing underground horizontally and, as a result, vertical components of magnetic field disturbances would become even smaller. Strictly speaking, this assumption requires that a three-dimensional structure be prepared and a relational expression be solved. In this case, current integrations must be performed over an infinite range and the conditions of the subterranean distribution of electric conductivity must be grasped, which is practically impossible.

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