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Measurement of the Coil Constant of the Large Helmholtz Coil Set of the West Pillar in the Calibration House

by

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1. Introduction

The large, rectangular, triaxial Helmholtz coil set installed of the west pillar in the calibration house(here in after referred to as the large coil set) is used for the calibration and performance tests of magnetometers of the observatory and other institutes. This large coil set is used, for example, to measure the sensitivity calibration values and axis orthogonality of a fluxgate magnetometer (FM) with a high level of accuracy. However, such measurement requires that the coil constants of individual coils and other basic values be known in advance. The coil constant, which represents the ratio of the current through a coil to the magnetic field at the center of the coil, is essential in order to generate an artificial magnetic field of a given magnitude with high accuracy. Since the large coil set is used to test many instruments, its coil constant is one of the values that, in effect, determine the accuracy of the Kakioka Magnetic Observatory's standards regarding magnetic fields.

Theoretically, the coil constant of a coil is determined purely by its geometric shape. Therefore, the coil constants calculated from the lengths of sides, intervals between coils, and other values measured by the coil manufacturer at the time of delivery were used for some time after the assemblage of the coil set. However, considering the subsequent improvement in the performance of FMs, it is doubtful whether these values are sufficient to ensure the accuracy required to test FMs with the large coil set. For example, FMs that are frequently used nowadays are required to measure magnetic field changes of about ±500 nT with an accuracy tolerance of 0.1 nT or less. In order to test instruments with these levels of accuracy, it is necessary to determine the coil constants with an accuracy tolerance of at most 0.02%, preferably less. Calculating the coil constants by actually measuring the coil geometry would require that the lengths of the sides of the large coil set, which are about 3 m each, be measured with an accuracy tolerance of about 0.6 mm in order to satisfy the requirements for the needed accuracy. In addition, the determination of the coil constants requires measuring the lengths of coils and intervals but also the relative distortions and twists between the sides with the same level of accuracy. For these reasons, it is very difficult to actually determine the coil constants by direct measurement of coil geometry. Even if we could determine the coil constants in this way, they may be subject to subtle changes caused by the subsequent distortions and transformations of the coil frame.

Due to these difficulties, methods for measuring coil constants have not been sufficiently studied despite their importance. In order to study with what level of accuracy coil constants can be determined using currently available tools, we performed a re-measurement of the coil constants of the large coil set by using nearly the same method as previously used by Koike *et al.* (1990). By comparing our results with theirs, we would like to provide an opportunity to discuss the most effective methods for measuring the coil constants.

2. Outline of the measurement method

The coil constants of the large coil set were measured by Koike *et al*. (1990) with an optical pumping magnetometer (OPM) manufactured by Mitsubishi Electric Corporation. The same magnetometer was used in our measurement. Our measurement method can be summarized as follows: The OPM measures total magnetic intensity and is said to fundamentally have the ability to measure total magnetic intensity over a wide measurement range without sensitivity calibration values used for instruments like FMs. We installed the OPM at the center of the large coil set, with electric currents of a certain magnitude running in positive and negative directions so as to generate artificial magnetic fields of the same size and opposite directions.

In the Terence electric currents artificial and natural magnetic fields are mixed and cause a change in the total magnetic intensity at the center of the coils as compared to the state without electric currents. From the change in total magnetic intensity caused by the electric currents running in positive and negative directions, we can calculate the magnitude of the artificial magnetic fields generated at the center of the coil set along the axis of the coils. Once the magnitude of the artificial field is determined, we can determine the coil constant by calculating the ratio of the artificial field to the electric currents.

Although this method can be employed to measure the coil constant of the north-south or vertical axis coils, it does not provide an accurate measurement of the constant of the east-west axis coils, since changes in the total magnetic intensity at the center of the coil set are very small compared to the magnitude of the artificial magnetic field oriented in this direction. Accordingly, Koike *et al*. (1990) calculated the coil constant of the north-south axis coils by the above method in order to determine the sensitivity calibration value of the H component of an FM, and then calculated the coil constant of the east-west axis coils from the ratio of the calibration value to the change in the magnetic field measured by the FM by applying an artificial magnetic field along the east-west axis using the H-component sensor of the FM. However, this method allows errors in the measurement of the coil constants of the north-south axis coils to be included in the coil constants of the east-west axis coils. To avoid this, we determined the coil constants of the east-west axis coils by independent as measured of total magnetic intensity.

Except for the east-west coil axis, our method is basically the same as the method used by Koike *et al*. (1990). However, we processed measured values more strictly in order to prevent measurement errors as much as possible. As an example, we will explain in detail how the constants of the H-component coils were determined. Suppose that the composite vectors F_+ and F_- created from total magnetic intensity (F_0) and artificial magnetic fields $+h$ and $-h$ are measured by the OPM installed at the center of the coil set, as shown in Figure 1. The following equation holds for the artificial magnetic field created by applying $+h$:

$$\begin{aligned} F_+^{\ 2} &= (H_0 + h)^2 + Z_{\perp 0}^{\ \ 2} + D_{\perp 0}^{\ \ 2} \\ &= F_0^{\ 2} + h^2 + 2H_0 h \end{aligned} \tag{1}$$

Similarly, we obtain the following equation for the artificial magnetic field created by applying $-h$:

$$F_-^{\ 2} = F_0^{\ 2} + h^2 - 2H_0 h \tag{2}$$

where H_0 is the component of the natural magnetic field along the coil axis and F_0 is the total magnetic intensity of the natural magnetic field, while $Z_{\perp 0}$ and $D_{\perp 0}$ are respectively the up-down and east-west * component values of the natural magnetic fields across the H axis at right angles to each other. By adding these two equations, we obtain

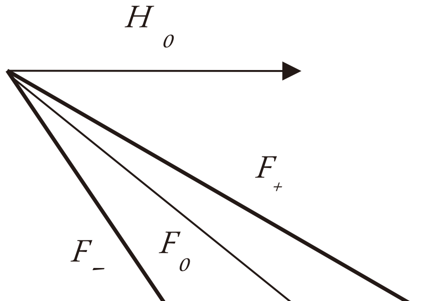

Fig. 1 Conceptual diagram showing artificial magnetic field in the case of running a current through the H-axis coil.

$$h^2 = \frac{F_+{}^2 + F_-{}^2}{2} - F_0{}^2 \tag{3}$$

From this equation, we can calculate h. In actuality, however, due to variations in the natural magnetic field during measurement, it is not possible to use the same H_0 values for equations (1) and (2). Actually, the following equations apply;

$$F_+{}^2 = F_{0+}{}^2 + h^2 + 2H_{0+}h \tag*{(1)'}$$

$$F_-{}^2 = F_{0-}{}^2 + h^2 - 2H_{0-}h \tag*{(2)'}$$

where F_{0+} and H_{0+} represent the F and H components of the natural magnetic field at the center of the coils, (which would have been observed at that point if no artificial magnetic field had been applied), during the positive magnetic field was being applied. Similarly, F_{0-} and H_{0-} represent those during the negative magnetic field was being applied (see Figure 2). Hence, h can be independently calculated from these equations by regarding equation (1)' or (2)' as a quadratic equation in h. Note, however, that there is a need to measure the new unknowns H_{0+} and H_{0-} in these equations by some indirect mean, because total magnetic intensity and component values cannot be measured at the same time at the center of the coils (as is expected when using an OPM for measurement). Therefore, in actual calculation, the H component at the center of the large coil set was calculated from the component value observed during the measurement of F at a point not affected by the magnetic field generated by electric currents through the coils. (In our measurement, we used the high sensitivity FM (90FM) in the basement variometer house as the reference point.) Also, instead of solving equations (1)' and (2)' separately, we combined these equations into a single quadratic equation, as shown below.

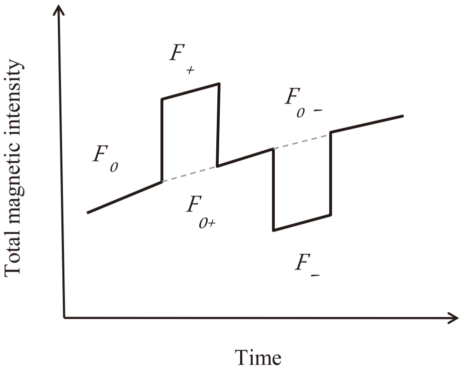

Fig. 2 Temporal variation in total magnetic intensity at the center of the coil set.

In our measurement, we applied both positive and negative magnetic fields for a continuous period and calculated averages in order to improve measurement accuracy. Thus, from equations (1)' and (2)', we obtain the following equations:

$$\overline{F_+{}^2 - F_{0+}{}^2} = h^2 + 2\overline{H_{0+}}h \tag{4}$$

$$\overline{F_-{}^2 - F_{0-}{}^2} = h^2 - 2\overline{H_{0-}}h \tag{5}$$

The overvars represents averages of measurements taken at different times. h has no overvar because an artificial magnetic field was assumed constant during the given period. Adding equations (4) and (5) yields the following quadratic equation:

$$h^2 + \alpha h + \beta = 0$$

$$\begin{aligned} \alpha &= \overline{H_{0+}} - \overline{H_{0-}} \\ \beta &= -\frac{1}{2}\left(\overline{F_+{}^2 - F_{0+}{}^2} + \overline{F_-{}^2 - F_{0-}{}^2}\right) \end{aligned} \tag{6}$$

Hence, h can be obtained by solving this quadratic equation,:

$$h = \frac{1}{2}\left(-\alpha \pm \sqrt{\alpha^2 - 4\beta}\right) \tag{7}$$

The term inside the root on the right-hand side of this equation is always positive. Supposing that the value of h is also sufficiently larger than α (artificial magnetic fields are larger in magnitude than variations in the natural magnetic field), the plus-minus sign on the right-hand side can be replaced by a plus sign (see Appendix 1). Hence,

$$h = \frac{1}{2}\left(-\alpha + \sqrt{\alpha^2 - 4\beta}\right) \tag*{(7)'}$$

This equation includes the component value of the magnetic field at the center of the coils on the right-hand side as α ($=\overline{H_{0+}} - \overline{H_{0-}}$). However, unlike when directly solving the equations (4) and (5), α is given in the form of a difference between two component values. Therefore, even if α is calculated indirectly from measurements taken by the 90FM in the basement variometer house, most of the error factors are eliminated, including magnetic field difference, instrumental errors, and differences in the properties of natural changes. Consequently, it is possible to calculate the value of α with sufficient accuracy if variations are accurately measured.

The argument above is based on the assumption that there are no instrumental errors in the OPM used for the measurements. Since the OPMs used for the actual measurement can produce errors compared with the values measured by the standard instrument (MO-PK(B)), an equation for β should be pron'ed assuming that the OPM and measured values always contain a constant instrumental error. If we denote the actual value measured by the OPM by F'(including the instrumental errors), the correction value for the instrumental errors by p and the true value by F ($= F'+p$), the following equation holds:

$$\beta = \beta' + \Delta\beta$$

$$\beta' = -\frac{1}{2}(\overline{F'^2_+ - F'^2_{0+}} + \overline{F'^2_- - F'^2_{0-}})$$

$$\Delta\beta = -p(\overline{F'_+} - \overline{F'_{0+}} + \overline{F'_-} - \overline{F'_{0-}})$$

Calculation of β requires the value measured by the 90FM, alike α, because it is impossible to measure both F_+ and F_{0+}, or F_- and F_{0-}, at the center of the large coil at the same time. Therefore, we calculated F_{0+} and F_{0-} from the F components created from the 90FM measurement (preliminary absolute values) (the F value measured by the 90FM during measurement with positive current is denoted by F_{FM+} and the F value measured during measurement with negative current by F_{FM-}.) For the calculation of these values, the term on the right-hand side of β', which represent a positive artificial magnetic field, for example, can be transformed as follows (to avoid complexity, measured value F' is redefined as F in the following equation):

$$\overline{F_+^2 - F_{0+}^2} = \overline{F_+^2 - (F_{FM+} + \Delta F_+)^2}$$

$$= \overline{F_+^2 - F_{FM+}^2} - 2\overline{\Delta F_+ F_{FM+}} - \overline{(\Delta F_+)^2}$$

where

$$\Delta F_+ = F_{0+} - F_{FM+}$$

which represents the difference between F values measured at the center of the coil and at the 90FM point in the basement variometer house (including instrumental errors of the OPM and the 90FM). The symbols with overbars represent the average values during measurement with positive currents. Although F values measured by the 90FM include instrumental errors, these errors eliminate each other in the process of the transformation of the equations. Similarly, F_{0-} can be eliminated by using F_{FM-} and ΔF_- from the terms representing measurement with negative current. Further assuming that the magnetic field difference between the center of the coil and the basement variometer house does not vary with time, ΔF_- and ΔF_+ can be substituted by a fixed constant ΔF, which can be calculated from values measured by the OPM and the 90FM during measurement with no artificial magnetic field. It is also possible to eliminate F_{0+} and F_{0-} from $\Delta\beta$ in exactly the same way by using F calculated from measurements by the 90FM. To summarize, we used the following equations in our measurement:

$$h = \frac{1}{2}\left(-\alpha + \sqrt{\alpha^2 - 4\beta}\right)$$

$$\alpha = \overline{H_{0+}} - \overline{H_{0-}}$$

$$\beta = \beta' + \Delta\beta \qquad (8)$$

$$\beta' = -\frac{1}{2}(\overline{F_+^2 - F_{FM+}^2} + \overline{F_-^2 - F_{FM-}^2}) +$$

$$\Delta F(\overline{F_{FM+}} + \overline{F_{FM-}}) + (\Delta F)^2$$

$$\Delta\beta = -p(\overline{F_+} + \overline{F_-} - \overline{F_{FM+}} - \overline{F_{FM-}} - 2\Delta F)$$

$$\Delta F = \overline{F_0} - \overline{F_{FM0}}$$

where F_{FM0} represents F in the basement variometer house measured by the 90FM during measurement with no electric currents through the coils. Terms with overvars represents averages during certain time periods. Note, however, that the time periods for average is differ between the terms and is one of the three time intervals (during positive current, negative current and no electric current).

Thus, h can be calculated from F_+, F_-, F_0 (by the OPM), F_+, F_-, F_0, p (instrumental errors contained in these values) and H_{0+}, H_{0-}, F_{FM+}, F_{FM-} and F_{FM0}. (calculated from measurements by the 90FM), H_{0+}, H_{0-}, F_{FM+}, F_{FM-} and F_{FM0}. (Instrumental error with regard to α, can be ignored because it represents the difference between measured values. Although we assumed that the instrumental error of the OPM is constant, it is doubtful whether this assumption is valid or not in reality. This issue will be discussed in detail later.

Once h is obtained as shown above, the coil constant can be determined from the currents running through the coil (i_H) by the following equation:

$$C_H = \frac{h}{i_H} \tag{9}$$

The coil constants of the Z-axis coil (vertical component) and the D-axis coil (east-west component) were calculated in the same way.

As mentioned in the next section, actual determination of coil constant were consisted of a number of pair measurements with positive and negative electric currents. We calculated one coil constant for each of measurements, and the averages of these constants ($\overline{C_H}$, $\overline{C_Z}$, $\overline{C_D}$) were used as the final measured values of the coil constants to be compared with the previous measurement results.

3. Actual measurement procedures

The MQM-100, a cesium (^{133}Cs) OPM manufactured by Mitsubishi Electric Corporation (identical to the total-force magnetometer used by Koike *et al*. (1990)) was used for measurement. The outline of the wire connections between the OPM and peripheral devices is shown in Figure 3. The OPM outputs a frequencies in proportion to the intensity of the magnetic field,which are multiplied fourhold with frequency multiplier(manufactured by Kokusai Electronics Corporation) with their waveforms reshaped and amplified (since frequencies are converted to magnetic field values by post-processing, there is no particular need to multiple frequencies fourfold, but we used the frequency multiplier because normal measurements could not be performed without it). The output frequencies were measured with a frequency counter, the TR5824, with a resolution of 1 Hz. Then, the output from the counter in BCD form were converted into serial data with a BCD-RS232C converter (manufactured by Technica Corporation) to be recorded on a PC every second. The frequency data recorded were converted into magnetic field values by calculation. The following expedient value previously used for the OPM of the KASMMER system was used as the conversion constant (equivalent to the so-called sensitivity calibration value):

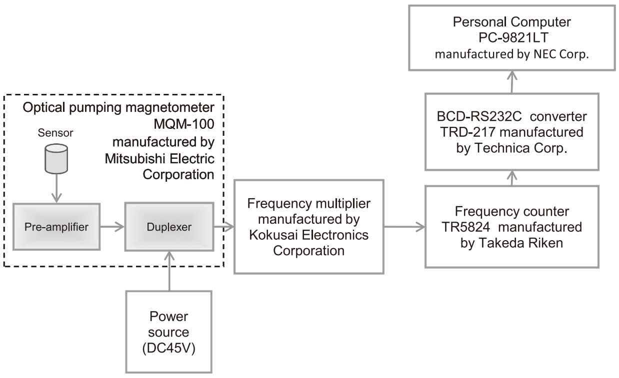

Fig. 3 Composition of devices for the measurement of the coil constants.

$$t = 0.2857142 = \frac{4}{14} \quad nT / Hz \tag{10}$$

(Recorded frequencies, which were already multiplied fourfold, were simply divided by 14.*) The resolution of the magnetic field values finally obtained was about 0.07 nT and their variation was about 0.1 nTp-p. The value in (10) above is no more than an expedient value used in the previous system (Sano, 1973). Errors resulting from the selection of this value will be discussed later. The direct current power source Model 2561 (S/N 00261) in the measurement processing house, manufactured by Yokogawa Electric Works, was used to supply electric currents to the large coil set. It was tested by the manufacturer in October 1997, just before our measurement.

Since α on the right-hand side of equation (7)' is ordinarily very small, the accuracy of the solution is almost completely determined by the accuracy of β. However, as can be seen from the equation involving β, β becomes larger and the magnitude of an artificial magnetic field is determined with greater accuracy, when the

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* In the former KASMMER system, magnetic field values were directly output in units of 0.1 nT by counting the number of waves for 1/1.4 seconds, with frequencies multiplied fourfold (Yanagihara *et al*., 1973).

change in total magnetic intensity affected by currents through the coils is larger. In our measurement, therefore, we used the largest artificial magnetic fields allowable for our purposes and measured the coil constants by using magnetic fields with two different intensities (approximately 5000 nT and 2500 nT). When such large magnetic fields are generated by the large coils, the effects on the environment around the calibration house — the measured values of Overhauser magnetometers in adjacent house in particular — cannot be ignored (Yamada *et al*., 1997). In order to avoid disturbance effects on the one-minute averages (from the 30th second to the next 29th second) of the Overhauser magnetometer, we made sure that all set of measurements with positive and negative currents were taken within a one-minute period between the 30th and the next 30th second. We also ensured that positive and negative currents were passed through the coils for the same duration in order to eliminate disturbance effects on the average. More specifically, currents were switched from one state to another as shown in Table 1. Therefore, the duration of positive, negative, and no currents within a one-minute period were the same (20 seconds). (Since currents were switched manually, we could not control artificial magnetic fields precisely in accordance with the time frame shown in the table. Therefore, values for the one-second periods before and after the switching of currents (35th, 55th, 05th and 25th seconds) were not used in the subsequent processing.)

To calculate β for a one-minute period, we calculated ΔF from the difference between the F values measured by the OPM during no currents through the coils and the F values calculated from the 90FM. We also calculated α for the periods of time when positive and negative currents through the coils from measurement taken by the 90FM during the same periods. In these calculations, we used the preliminary absolute values of the 90FM for the H and Z components without change with regard to the H-axis and Z-axis coils. The reason for using the unaltered H component (horizontal component) values is that the H-coil axis is oriented almost exactly in the direction of magnetic north (about 6.5° west from the true north). With regard to the D-axis coils, we used the direction calculated by Uwai *et al*. (1990) to convert the H component of the 90FM into the D-axis component. Since α is ordinarily very small compared with β, minor errors in the sensitivity calibration values of the 90FM or in the conversion into coil-axis components do not have any significant effect on the calculation of coil constant values. In particular, conditions were magnetically relatively stable during the period when the measurements were performed. Accordingly, variations in the natural magnetic field during a one-minute period were minor and the contributions of α to the calculation of the coil constants were very small.

Table 1 Procedure for switching coil currents from the 30th second to the next 30th second.

30 to 35 sec.	35 to 55 sec.	55 to 05 sec.	05 to 25 sec.	25 to 30 sec.
0	+	0	-	0

Symbols 0, + and - respectively represent a period with no currents running through the coil, a period with positive currents and a period with negative currents.

We obtained one constant from a one-minute measurement and consecutively performed measurements for 10 minutes for the H and Z axes and for 20 minutes for the D axis, and then averaged the individual constants in order to obtain the final coil constant values. We increased the number of measurements for the D axis because variations in total magnetic intensity produced at the center of the coils by an artificial magnetic field along the D axis was small compared with variations along the H and Z axes. This reduced the accuracy in determining the coil constants for the D axis by a single measurement. Approximate values (precise values vary from time to time depending on absolute magnetic field values) of variations in total magnetic intensity at the center of the coils during the measurement are presented in Table 2 as a reference.

The instrumental error of the OPM measured 18.8 nT. However, since the cable between the sensor and the pre-amplifier was short, there is a possibility that the pre-amplifier caused a disturbance effects on the magnetic fields (with an intensity of about 1 nT at maximum). The directional differences between the sensors of different devices (differents of 2 nT at maximum)

Table 2 Approximate variations (nT) in total magnetic intensity at the center of the coil set caused by positive/negative currents running through the coil.

	North-south	East-west	Vertical
Measurement with a magnetic field of about 5000 nT			
Positive	~ 3200	~ 200	~ 3600
Negative	~ -3000	~ 270	~ -3500
Measurement with a magnetic field of about 2500 nT			
Positive	~ 1600	~ 52	~ 1800
Negative	~ -1600	~ 83	~ -1800

A “positive current” is a current that generates a northward, eastward and downward magnetic field for the respective coil.

Table 3 Results of the measurements of coil constants in nT/mA.

Measurement with a magnetic field of about 5000 nT	North-south	East-west	Vertical
Coil constant	11.230	11.639	10.869
Difference with the current value	+0.008	+0.011	0.000
Standard deviation (σn)	0.0004	0.0003	0.0004
No. of measurements performed	10	20	10

Measurement with a magnetic field of about 2500 nT	North-south	East-west	Vertical
Coil constant	11.231	11.641	10.869
Difference with the current value	+0.009	+0.013	0.000
Standard deviation (σn)	0.0007	0.0012	0.0009
No. of measurements performed	10	19	10

Currently used value (1990)	11.222	11.628	10.869
Value calculated from the size	11.232	11.633	10.866

were clarified. Directional differences are unlikely to have had any major effects on our measurements, since the sensors were oriented in the same direction when measuring the instrumental errors and the coil constants. However, there is a need to check this point once again. For the calculation of instrumental errors,we used the equation (10) to calculate the magnetic field values of the OPM, as the measurment of the coil constants.

4. Measurement results and considerations

Measurements were performed on December 4 (with a magnetic fields of 5000 nT) and December 9 (with 2500 nT), 1997. Table 3 shows the results. The current standard values (measured by Koike *et al.* (1990)) and the values calculated from the size of the coils (adopted before the measurement by Koike *et al.*) are also shown as references. These results indicate the following observations:

(1) Standard deviations in the measurement with magnetic fields of 5000 nT and 2500 nT were sufficiently small. Variations in the measurement with magnetic fields of 5000 nT were on the order of 0.0001 nT/mA, and variations in the measurement with magnetic fields of 2500 nT were slightly larger. Variations were sufficiently small to determine the coil constant within an accuracy of 0.001. In the measurement with magnetic fields of 2500 nT, the accuracy of the constants of the east-west axis coils was lower than those of the north-south and vertical axis coils. This is due to the fact that variations in total magnetic intensity caused by the switching of currents were particularly small when measuring the east-west component of magnetic fields of 2500 nT.

(2) The constants of the north-south and east-west axis coils measured with magnetic fields of 2500 nT are slightly larger than the constants of the same coils measured with 5000 nT fields. These differences are larger than the standard deviations in the measurement and are likely to be statistically significant.

(3) The measured coil constant of the vertical axis coils were exactly the same as the current standard values (Koike *et al.*, 1990). However, our values for the north-south and east-west axis coils were larger by about 0.01nT/mA than the current standard values. These differences are not negligible current requirement of coil constants accuracy.

One of the objectives of our coil constants measurement was a stability check of the current coil constant measured in 1990. If the differences in (3) above are valid, there were considerable changes in the coil constants of the north-south and east-west axis coils after the last measurements. These changes, which were on the order of 0.1%, are sufficiently large to effect on the measurement accuracy of sensitivity calibration values (ordinarily determined to three decimal digits) of the FM. Have the coil constants truly changed to such degree?

The differences in (2) above raise the most serious concerns about our measurement, because the same results should be obtained whether the applied magnetic fields were 5000 nT or 2500 nT. We need to clarify the causes of these differences and confirm the reliability of our measurement before discussing the differences in (3). We will focus on potential error factors of the coil constant values in our measurement and data processing.

a) Errors in the current from the constant-current source

Errors in the electric current values directly affect the obtained coil constant values. Assume that the electric current values changed during measurement. Changes in the coil constants of three axes were different rather than the same amount of increase or decrease as the same time. More specifically, the same values were obtained for the vertical axis coils in both measurements, while the results obtained for the north-south and east-west axis coils with magnetic fields of 2500 nT were larger than the results with 5000 nT fields and the differences were larger for the east-west axis coil than for the north-south axis coil. We consecutively performed measurements for the north-south, vertical, and east-west axis coils, in this order, without intervals and performed measurements with 5000 nT fields and 2500 nT fields on different days. Therefore, the difference in the variation among three axes implies that: Possible changes in electric current values were not long-term one such as differences between two days, but changes within a single day, during the respective periods of measurement of the north-south, vertical, and east-west axis coils. However, such short-term changes would have caused larger standard deviations than those in Table 3 and would have produced variations of the same level as differences in (2) during the measurement. Therefore, the differences in (2), at least, cannot be explained by changes in the electric current. Neither can the differences in (3), which also are different among axes, be explained by changes in electric current.

b) Measurement errors of peripheral measuring devices

Even if there were errors in the output of the frequency counter and amplifier used for the measurement within the OPM, these errors cannot be the cause of the differences in (2) and (3) for the same reason as explained above.

c) Errors in the F values calculated from measurements taken by the 90FM

As F values at the center of the large coils are estimated from measurements taken by the 90FM and used as constants, there is a possibility that the magnetic field difference may vary over time depending on the location. However, in our measurement, the average of magnetic field differences observed when no currents was running through the coils was calculated for each one-minute period and used as ΔF. Therefore, variations during these time periods are negligible.

d) Measurement errors of the OPM

OPMs have various error factors by their very nature (see Sano (1972) for example). Shifts in temperature, angle, and light intensity are known as error factors resulting from the mechanical structure of magnetometers. These are considerable uncertainties when using an OPM for the absolute total magnetic intensity measurement. These error factors are usually corrected as instrumental errors. However, our measurement of instrumental errors itself may include errors, as has already been mentioned. As the exact extent of variations in these instrumental errors remains unknown, we re-calculated the coil constants, partly in order to check the results of our calculation, by varying the instrumental error from the measured value (18.8 nT) by ±5 nT. However, the effects of these variations were minor, with only the last decimal digit of the coil constants (third decimal digit) being affected. As is estimated in Appendix 2, variations in the instrumental errors cannot cause the differences mentioned in (2) unless they are exceptionally large. Therefore, it is at least clear that minor variations are not sufficient to provide explanations for the differences in (2) or (3).

Meanwhile, considerably wider range of measured magnetic field than is ordinary observation is another error factors resulting from the use of the OPM in our measurement of the coil constants.

The intensity range from 40000 nT to 50000 nT needs to be measured during measurements with a magnetic field of 5000 nT. In contrast, we carry out instrumental error measurements with magnetic fields of ordinary intensity (around 46000 nT at the Kakioka Magnetic Observatory). Instrumental errors observed under these condition are not be applicable in such a wide range. (Since not much attention is paid to this matter, we provided a detailed description in Appendix 2.) However, although these errors have the smallest effects on the constants of the east-west axis coils, which had small magnetic field variations, errors in (2) are the largest for the east-west axis coils. Therefore, the differences in (2) cannot be explained by the measurement errors of the OPM. As is estimated in Appendix 2, these errors are likely to be very small with regard to the east-west axis coil.

5. Conclusion

Our measurement enabled us to calculate the coil constants with sufficient accuracy, at least with regard to individual measurements. However, results were different depending on the artificial magnetic fields used for measurement. The cause of these differences remains unknown. The coil constant values that we obtained were markedly different from those measured by Koike *et al.* (1990). Differences between these two sets of values are not negligible considering how these constants are ordinarily used. Therefore, there is a need to improve the measurement method in order to obtain more reliable values.

As was mentioned at the beginning of this paper, the coil constants of the large coil set are used as standards to determine the constants of various magnetometers. Whether or not the values obtained in our measurement can be used without modification remains an open question. Our measurement at least made us realize the difficulties involved in the measurement of the coil constants and the level of reliability required. We would be pleased if our research provided a foothold for establishing a method for checking constant values with a high level of accuracy.

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Appendix

Appendix 1: Root sign in equation (7)

By transforming equation (7) using equations (4) and (5), we obtain

$$\begin{aligned}\alpha^2-4\beta &= \left(\overline{H_{0+}}-\overline{H_{0-}}\right)^2+4\left(\overline{H_{0+}}-\overline{H_{0-}}\right)h+4h^2\\ &= \left\{\left(\overline{H_{0+}}-\overline{H_{0-}}\right)+2h\right\}^2\\ &= \left(\alpha+2h\right)^2 \geq 0\end{aligned}$$

Therefore, the terms inside the root sign in equation (7) never become negative. A similar transformation of equation (7) when the plus-minus sign is negative yields the following result:

$$h=\frac{1}{2}\left(-\alpha-\left|\alpha+2h\right|\right)$$

Assuming $h>0$ and $h>>|\alpha|$, we obtain $h=\frac{1}{2}\alpha$, which is completely impossible.

Appendix 2: A measurement error of the OPM due to differences in the range of measurement

The relationship between the oscillation frequency of an optical pumping magnetometer (f) and the magnetic field used for measured (F) can be represented by the following equation (Sano, 1973):

$$f=aF+bF^2$$

By solving this equation for F and by ignoring higher order terms, we obtain the following equation:

$$F=\frac{1}{a}f-\frac{b}{a^3}f^2 \tag{A-1}$$

Noda and Ota (1970) defined the values of these coefficients regarding Cs^{133} as follows:

$$\begin{aligned}a &= 349{,}869\ Hz/gauss &&= 3.49869\ Hz/nT\\ b &= 26.8\ Hz/gauss^2 &&= 2.68\times10^{-9}\ Hz/nT^2\end{aligned} \tag{A-2}$$

(According to Ochi (1968), the values of these coefficients regarding Rb^{85} are defined as follows:

$$\begin{aligned}a &= 466{,}734.8\ Hz/gauss &&= 4.667348\ Hz/nT\\ b &= 359.4\ Hz/gauss^2 &&= 3.594\times10^{-8}\ Hz/nT^2\end{aligned}$$

Therefore, in weak magnetic field (as in our measurement) and limited measurement range, the coefficient for the quadratic term is very small compared with the coefficient for the linear term and the relationship between oscillation frequencies and magnetic fields can be approximated to be linear. Based on this assumption, the value a_0=3.5 Hz/nT is used as a practical experiential value in the processing of measurements taken by the OPM of the former KASMMER system. The following equation, which adopts this value and adds the term c that represents instrumental errors caused by differences in measurement environment and measuring devices, was used in the processing:

$$F=\frac{1}{a_0}f+c \tag{A-3}$$

We used the same value for a_0 when processing the results of our measurement to calculate the constant values, However, the results of equations (A-1) and (A-3) differ if the measurement range is wider. The difference will appear as a variation in the instrumental errors in equation (A-3).

To show this, Figure A-1 was drawn by assuming the magnetic field given by equation (A-1) and constant (A-2) as the true magnetic field and by calculating how magnetic errors (instrumental errors) represented by f/a_0 caused by the same oscillation frequency vary depending on the magnetic fields for measurement. The figure shows that instrumental errors vary by about 3 nT within a magnetic field range from 40000 to 50000 nT.

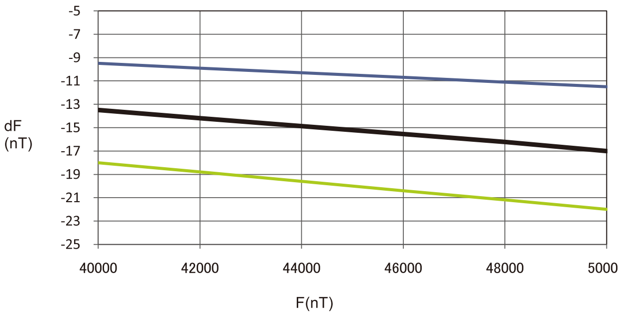

Fig. A-1 Change in the instrumental error of the OPM in relation to the intensity of magnetic fields to be measured. The thick line shows the change calculated by using the constant (A-2). The upper (lower) thin line was drawn by increasing (decreasing) the value of a in (A-2) by 0.01% and the value of b in (A-2) by 10% in order to estimate the effects of errors in those constants.

We will estimate the error of the calculated coil constants caused by variation of instrumental errors depending on the measurement range. To simplify the argument, we will start with equation (3) by assuming that there is no variation in the natural magnetic field within the given measurement period. By differentiating equation (3), we obtain

$$2hdh = F_+ dF_+ + F_- dF_- - 2F_0 dF_0 \tag{A-4}$$

where dF_+, dF_- *and* dF_0 are instrumental errors of F_+, F_- and F_0, respectively. Let these instrumental errors be denoted as

$$dF_+ = dF_0 + dp_+$$
$$dF_- = dF_0 + dp_-$$

where dp_+ and dp_- represent changes in the instrumental errors of the OPM caused by positive and negative magnetic fields, respectively. Rewriting F_+ and other variables by using equations (1) and (2) yields the following:

$$F_\pm = F_0 \sqrt{1 + \frac{h^2 \pm 2hH_0}{F_0^2}} \approx F_0 (1 \pm \frac{hH_0}{F_0^2})$$

Therefore, equation (A-4) can be written as

$$2hdh = F_0 (1 + \frac{hH_0}{F_0^2}) dp_+ + F_0 (1 - \frac{hH_0}{F_0^2}) dp_- \tag{A-5}$$

The dF_0 term is eliminated from the above equation, which means that a certain part of instrumental errors (offset) has almost no effect on the coil constant $\frac{dh}{i_H}$. By linear approximation of changes in instrumental errors, as shown in Figure A-1, the following equation holds valid:

$$dp_+ = -dp_- = dp$$

Hence, from equation (A-5), we obtain

$$dh = \frac{H_0}{F_0} dp \tag{A-6}$$

Therefore, variation in instrumental error directly contributes to error in h. Our estimation of the coil constant error based on the above equation indicates that h of 5000 nT and dp of −1 nT the error in the constants of the H and Z coils of around −0.0015 nT/mA and a negligible error in causes the constants of the D coil. The constants of the H and Z coils obtained in our measurement include such errors. However, these errors cannot explain the differences between the results with 5000 nT magnetic fields and those with 2500 nT magnetic fields because the variation range in total magnetic intensity at the center of a coil as well as dp expands roughly in proportion to the coil's electric current, and the overall effects on the coil constant do not change.